PG-AS-417	**MMSS–11**

P.G. DEGREE EXAMINATION — JULY, 2022.

Mathematics

(From CY – 2020 onwards)

First Semester

ABSTRACT ALGEBRA

Time : 3 hours | Maximum marks : 70

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following each in 300 words.

1. Show that $N(a)$, normalizer of an element a in a group G, is a subgroup of G.

2. Let G be a group and suppose that G is the internal direct product of $N_1, N_2, ..., N_n$. Let $T = N_1 \times N_2 \times ... \times N_n$. Show that G and T are isomorphic.

3. State and prove the Eisenstein criterion.

4. Define fixed field of a group and show that it is a subfield of K.

5. Show that S_n is not solvable for $n \geq 5$.

6. If G is a finite group, p is a prime and $p^n \mid o(G)$ but $p^{n-1} o(G)$, then show that any two subgroups of G of order p^n are conjugate.

7. If $f(x), g(x)$ are two nonzero elements of $F[x]$, then show that
$\deg(f(x)g(x)) = \deg f(x) + \deg g(x)$.

8. Show that for every prime p and every positive integer m there exists a field having p^m elements.

SECTION B — ($3 \times 15 = 45$ marks)

Answer any THREE of the following each in 1000 words.

9. State and prove Cauchy's theorem.

10. Show that two abelian groups of order p^n are isomorphic if and only if they have the same invariants.

11. Show that any two splitting fields of the same polynomial over a given field F are isomorphic by an isomorphism leaving every element of F fixed.

12. If F is of characteristic 0 and if a, b are algebraic over F, then show that there exists an element $c \in F(a, b)$ such that $F(a, b) = F(c)$.

13. State and prove the Wedderburn's theorem on finite division rings.

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PG–AS-418	MMSS–12

P.G. DEGREE EXAMINATION — JULY 2022.

Mathematics

(From CY – 2020 Onwards)

First Semester

ADVANCED CALCULUS

Time : 3 hours Maximum marks : 70

SECTION A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

1. Using basic mean value theorem, find the numbers θ_1 and θ_2 if $f(x,y) = x^2 + 3xy + y^2$, $a = b = 0$, $\Delta x = 1, \Delta y = -1$.

2. If $u = x + y + z$, $uv = y + z uvw = z$, then show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1}{u^2 v}$.

3. Divide 24 into three positive numbers *x, y, z* such that xy^2z^3 is maximum.

4. Evaluate by Green's theorem $\int_s e^{-x}(\sin y\,dx + \cos y\,dy)$, where S is the rectangle with verties *(0, 0),* $(\pi,0)$, $\left(\pi,\frac{\pi}{2}\right)$ and $\left(0,\frac{\pi}{2}\right)$.

5. Evaluate $\iint (y-x)dxdy$ over the region R_{xy} in the *xy*-palne bounded by the straight lines $y = x - 3$, $y = x + 1$, $3y + x = 5$, $3y + x = 7$.

6. Find $\frac{du}{dx}$ if $u = \sin(x^2 + y^2)$ where $a^2x^2 + b^2y^2 = c^2$.

7. Show that the function $f(x, y, z) = x^2 + y^2 + 3z^2 - xy + 2xz + yz$ has a relative minimum at *(0,0,0).*

8. Evaluate the integral $\int_\Gamma xdx + ydy + zdz$ where Γ is the circle $x^2 + y^2 + z^2 = a^2, z = 0$.

SECTION B — $(3 \times 15 = 45$ marks)

Answer any THREE of the following.

9. If $u = \log\left(\dfrac{x^2 + y^2}{\sqrt{x} + \sqrt{y}}\right)$, then prove that

(a) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{3}{2}$, and

(b) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = -\dfrac{3}{2}$

10. State and prove the inverse function theorem.

11. State and prove Taylor's theorem for functions of two variables.

12. Verify Gauss theorem for $\iint_s (4x\cos\alpha - 2y^2\cos\beta + z^2\cos\gamma)dS$, where S is the region bounded by $x^2 + y^2 = 4$, $z = 0$, $z = 3$ and α, β, γ are the angle between the exterior normal to the positive x-axis, y-axis and z-axis respectively.

13. Verify Stroke's theorem for the integral $\int_\Gamma ydx + zdy + xdz$, where Γ is the boundary of the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$.

PG-AS-419	**MMSS-13**

P.G. DEGREE EXAMINATION –
JULY 2022.

Mathematics

(From CY – 2020 Onwards)

First Semester

REAL ANALYSIS

Time : 3 hours Maximum marks : 70

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following.

1. Show that continuous image of a compact metric space is compact.

2. Does the limit of the integral is equal to the integral of the limit? Justify.

3. Show that there exists a non-measurable set.

4. State and prove the Lebesgue's monotone convergence theorem.

5. State:

(a) Lebesgue decomposition theorem.

(b) Riesz representation for L^1.

6. Let $f \in R$ on $[a, b]$ and if there is a differentiable function F on $[a, b]$ such that $F' = f$, then show that $\int_a^b f(x)dx = F(b) - F(a)$.

7. Show that every interval is measurable.

8. Show that $\int_0^\infty \frac{dx}{\left(1+\frac{x}{n}\right)^n \frac{1}{x^n}} dx = 1$.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE of the following.

9. Let f be a continuous mapping of a compact metric space X into a metric space Y. Show that f is uniformly continuous.

10. Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, then show that $\{f_n\}$ converges uniformly on $[a, b]$, to a function f, and

$f'(x) = \lim_{n\to\infty} f_n'(x) \; (a \le x \le b)$.

11. Show that the following statements are equivalent.

(a) f is a measurable function.

(b) $\forall \alpha, [x : f(x) \geq \alpha]$ is measurable.

(c) $\forall \alpha, [x : f(x) < \alpha]$ is measurable.

(d) $\forall \alpha, [x : f(x) \leq \alpha]$ is measurable.

12. State and prove the Lebesgue's dominated convergence theorem.

13. State and prove the Radon-Nikodym theorem.

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PG-AS-420	MMSSE-1

P.G. DEGREE EXAMINATION – JULY, 2022.

Mathematics

(From CY – 2020 onwards)

First Semester

DIFFERENTIAL GEOMETRY

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions each in 300 words.

All questions carry equal marks.

1. State and prove Serret – Frenet formulae theorem.

2. If θ is the angle at the point (u,v) between the two directions given by

   $Pdu^2 + 2Qdudv + Rdv^2 = 0$ then prove that $\tan\theta = \dfrac{2H\left(Q^2 - PR\right)^{\frac{1}{2}}}{ER - 2FQ + GP}$.

3. Prove that, on the general surface, a necessary and sufficient condition that the curve $v = c$ be geodesic is $EE_2 + FE_1 - 2EF_1 = 0$.

4. If K_n is the normal curvature of a curve at a point on a surface then prove that $K_n = \dfrac{Ldu^2 + 2Mdudv + Ndv^2}{E\,du^2 + 2Fdudv + Gdv^2}$ where E, F and G are first fundamental coefficients and

$$L = N.r_{11}\,,\ M = N.r_{12},\ N = N.r_{22}\,.$$

5. Prove that the only compact surfaces whose Gaussian curvature is positive and mean curvature constant are sphere.

6. Find the length of the curve given as the intersection of the surfaces

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1\,,\ x = a\cosh\left(\dfrac{z}{a}\right)$ from the point $(a,\ 0,0)$ to the point $(x,\ y,\ z)$.

7. Derive Canonical geodesic equations.

8. Prove that the principal curvature are given by the roots of the equation $\kappa^2\left(EG - F^2\right) - \kappa\left(En + GL - 2FM\right) + LN - M^2 = 0$.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions each in 1,000 words.

All questions carries equal marks.

9. State and prove Fundamental existence theorem for space curves.

10. Prove that the first fundamental form of a surface is a positive definite quadratic form in *du, dv*.

11. Prove that $u = u(t)$, $v = v(t)$ on a surface $\vec{r} = \vec{r}(u, v)$ is a geodesic if and only if the principal normal at every point on the curve is normal to the surface.

12. State and prove Rodrique's formula theorem.

13. State and prove Hilbert's theorem.

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PG–AS–421	MMSSE–2

P.G. DEGREE EXAMINATION — JULY, 2022.

Mathematics

(From CY – 2020 Onwards)

First Semester

PROGRAMMING in C++

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions. Each in 300 words

1. Explain the applications of OOPs.

2. Write a C++ program to find maximum of two numbers using inline functions.

3. What is copy constructor? When it is used implicitly for what purpose?

4. Give a programming example that overloads = = operator with its use.

5. Show the use of multiple inheritance with the help of proper programming example.

6. What do you mean by type conversion? Give an example of basic to object conversion

7. Explain the various operators that are available on C++.

8. Write a program in C++ that checks whether the given string is palindrome or not.

PART B — ($3 \times 15 = 45$ marks)

Answer any THREE questions. Each in 1000 words

9. Explain with an example the control statements in C++.

10. What is a friend function? What are the merits and demerits of using the friend function?

11. What is Static Member Functions? What are the features of static data member?

12. (a) What is operator overloading? List out the rules to overload a binary operator.

    (b) Write C++ program to add two vectors using + operator overloading.

13. Write a C++ program demonstrating use of the pure virtual function with the use of base and derived classes

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PG-AS-422	MMSS-21

P.G. DEGREE EXAMINATION – JULY, 2022.

Mathematics

(From CY – 2020 onwards)

Second Semester

APPLIED MECHANICS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions each in 300 words.

All questions carries equal marks.

1. Find the kinetic energy of a rigid body with a fixed point.

2. Prove that the rate of change of the angular momentum of a system about a point, either fixed or moving with the mass center, is equal to the total moment of the external forces about that point.

3. Discuss cuspidal motion of a top.

4. Derive Lagrange's equations motion for Implusive motion.

5. Explain poisson brackets.

6. A rectangular plate spins with constant angular velocity W about a diagonal. Find the couple which must act on the plate in order to maintain this motion.

7. Explain the Bilinear invariant.

8. Explain the motion of a rigid body with a fixed point under no forces using analytic method.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions each in 1000 words.

All questions carries equal marks.

9. Find the angular momentum of a rigid body.

10. Explain the general motion of a rigid body in methods of dynamics in space.

11. Discuss the general motion of a top.

12. Derive Hamilton's equations of motion.

13. Explain Hamilton's principle.

PG–AS-423 **MMSS–22**

P.G. DEGREE EXAMINATION —
JULY 2022.

Mathematics

(From CY – 2020 onwards)

Second Semester

COMPLEX ANALYSIS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions.

All questions carries equal marks.

1. State and prove Local Mapping theorem.

2. State and prove Rouche's Theorem.

3. State and prove Mittag-Leffler theorem.

4. State and prove Harnack's principle.

5. Show that non constant elliptic function has equally many poles as it has zeros.

6. If u_1 and u_2 are harmonic functions in a region Ω then prove that $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$ for every cycle γ which is homologous to zero in Ω.

7. State and prove Arzela's Theorem.

8. State and prove Cauchy's Integral formula.

PART B — $(3 \times 15 = 45$ marks)

Answer any THREE questions.

All questions carries equal marks.

9. Suppose the $\varphi(\zeta)$ is continuous on the arc γ. Then prove that the function

$F_n(z) = \int_\gamma \frac{\varphi(\zeta)\,d\zeta}{(\zeta - z)^n}$ is analytic in each of the regions determined by γ, and its derivative is $F_n'(z) = nF_{n+1}(z)$

10. If $pdx + qdy$ is locally exact in Ω then prove that $\int_\gamma pdx + qdy = 0$ for every cycle $\gamma \sim 0$ in Ω.

11. Derive

    (a) Jensen's formula and

    (b) Poisson-Jensen formula.

12. State and prove the Riemann mapping theorem.

13. State and prove existence and uniqueness theorem on canonical basis.

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PG-AS-424	**MMSS-23**

P.G. DEGREE EXAMINATION — JULY 2022.

Mathematics

(From CY – 2020 onwards)

First Year – Second Semester

LINEAR ALGEBRA

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE questions in 300 words.

All Questions carries equal marks

1. Let V and W be vector spaces over the field F and let T be a linear transformation from V into W. Suppose that V is finite-dimensional then prove that

$$rank(T) + nullity(T) = \dim V$$

2. Let F be a field and a. be a linear algebra with identity over F. Suppose f and g are polynomials over F, α is an element of a and that c belongs to F. Then prove that

(a) $(cf+g)(\alpha)=cf(\alpha)+g(\alpha)$

(b) $(f\,g)(\alpha)=f(\alpha)\,g(\alpha)$

3. Let T be a linear operator on an n-dimensional vector space V. Prove that the characteristic and minimal polynomials for T have the same roots, except for multiplicities.

4. Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is diagonalisable if and only if the minimal polynomial for T has the form $p=(x-c_1)\ldots(x-c_k)$ where $c_1,\ldots\ldots,c_k$ are distinct elements of V.

5. Let F be a field and let B be an $n\times n$ matrix over F. Then prove that B is similar over the field F to one and only one matrix which is in the rational form.

6. Let V be a vector space over the field F; Let U,T_1 and T_2 be linear operators on V; Then prove that

$U(T_1+T_2)=UT_1+UT_2$ and $(T_1+T_2)U=T_1U+T_1U$

7. Let F be a field of characteristic zero and f is polynomial over F with $\deg f \le n$. Then prove that the scalar c is a root of f of multiplicity r if and only if

$$\left(D^k f\right)(c) = 0, 0 \le k \le r-1$$
$$\left(D^k f\right)(c) \ne 0$$

8. Let W be an invariant subspace for T. Prove that the characteristic polynomial for the restriction operator T_w divides the characteristic polynomial for T and the minimal polynomial for T_w divides the minimal polynomial for T.

PART B — (3 × 15 = 45 marks)

Answer any THREE questions in 1000 words.

All Questions carries equal marks.

9. Let V be an n-dimensional vector space over the field F. and let W be an m-dimensional vector space F. Then prove that the space $L(V,W)$ is finite-dimensional and has dimension mn.

10. State and prove Taylors formula theorem.

11. State and prove Caylay-Hamilton theorem.

12. Let $\mathcal{F}$ be a commuting family of triangulable lineaer operators on V. Let W be a proper subspace of V which is invariant under $\mathcal{F}$. Then prove that there exist a vector α in V such that

(a) α is not in W

(b) for each T in $\mathcal{F}$, the vector T_α is in the subspace spanned by α and W.

13. State and prove that Cyclic Decomposition theorem.

PG–AS-425	MMSSE–3

P.G. DEGREE EXAMINATION — JULY 2022.

Mathematics

(From CY – 2020 onwards)

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 hours Maximum marks : 70

PART A — ($5 \times 5 = 25$ marks)

Answer any FIVE of the following.

1. Form a partial differential equation by elimination f from $z = xy + f\left(x^2 + y^2 + z^2\right)$.

2. Classify the following partial differential equations.

   (a) $\frac{\partial^2 u}{\partial x^2} + 4\left(\frac{\partial^2 u}{\partial x \partial y}\right) + 4\frac{\partial^2 u}{\partial y^2} = 0$

   (b) $xyr - \left(x^2 - y^2\right)s - xyt + py - qx = 2\left(x^2 - y^2\right)$

3. Show that the family of right circular cones $x^2 + y^2 = cz^2$, where c is a parameter, forms a set of equipotential services.

4. Obtain d'Alembert's solution of the one-dimensional wave equation.

5. Find the temperature in a sphere of radius a when its surface is maintained at zero temperature and its initial temperature is $f(r,\theta)$

6. Find the general solution of the differential equation $x^2 \frac{\partial z}{\partial x} - y^2 \frac{\partial z}{\partial y} = (x+y)z$

7. Find a particular integral of the equation $(D^2 - D')z = e^{x+y}$

8. A rigid sphere of radius a is placed in a stream of fluid whose velocity of the undistributed state is V. Determine the velocity of the fluid at any point of the disturbed stream.

PART B — (3 × 15 = 45 marks)

Answer any THREE of the following.

9. Prove that the general solution of the linear partial differential equation $Pp + Qq = R$ is $f(u,v) = 0$ where f is an arbitrary function and $u(x,y,z) = c_1$ and $v(x,y,z) = c_2$ form a solution of the equations $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

10. Reduce the one-dimensional wave equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial^2 y} = 0$ to canonical form.

11. Discuss the Dirichlet's problem for a sphere and obtain its solution.

12. A tightly stretched string of length l has its ends fastened at $x = 0$ and $x = l$. The midpoint of the string is pulled to a height h and then released from rest in that position. Obtain an expression for the displacement of the string at any subsequent time.

13. Determine the Green's function for the thick plate of infinite radius bounded by the parallel planes z = 0 and z = a.

PG–AS-426	MMSSE –4

P.G. DEGREE EXAMINATION — JULY 2022.

Mathematics

(From CY – 2020 onwards)

Second Semester

MATHEMATICAL STATISTICS

Time : 3 hours Maximum marks : 70

SECTION A — (5 × 5 = 25 marks)

Answer any FIVE of the following each in 300 words.

1. If X_r and X_s are the r^{th} and s^{th} random variables of a random sample of size n drawn from the finite population $\{c_1, c_2, \ldots c_N\}$, then show that $\operatorname{cov}(X_r, X_s) = -\frac{\sigma^2}{N-1}$.

2. Suppose that 100 high-performance tires made by a certain manufacture lasted on the average 21,819 miles with a standard deviation of 1,295 miles. Test the null hypothesis $\mu = 22{,}000$ miles against the alternative hypothesis $\mu < 22{,}000$ miles at the 0.05 level of significance.

3. Obtain maximum likelihood estimates of the parameter α, β and σ

4. Write a short note on latin square design of experiments.

5. Let p = 2 and n = 1, and consider the random vector $X = \{X_1, X_2\}$. The discrete random variable X_1 have the following probability function. Find $E(X)$.

x_1	–1	0	1
$p_1(x_1)$	0.3	0.3	0.4

6. If $X_1, X_2, \ldots X_n$ constitute a random sample of size n from Bernoulli population, then show that $\hat{\theta} = \frac{x_1 + x_2 + \ldots x_n}{n}$ is a sufficient estimator of the parameter θ.

7. A random sample of size n from a normal population with $\sigma^2 = 1$ is to be used to test the null hypothesis $\mu = \mu_0$ against the alternative hypothesis $\mu = \mu_1$, where $\mu > \mu_0$. Use the Neyman-Pearson lemma to find the most powerful critical region of size α.

8. If the joint density function of X_1, X_2 and X_3 is given by

$$m(x_1, x_3) = \begin{cases} \left(x_1 + \frac{1}{2}\right) e^{-(x_3)}, & for\, 0 < x_1 < 1, x_3 > 0 \\ 0 & , \quad elsewhere \end{cases}$$

Find the regression equation of X_2 on X_1 and X_3.

SECTION B — (3 × 15 = 45 marks)

Answer any THREE of the following each in 1000 words.

9. (a) In 16 test runs the gasoline consumption of an experimental engine had a standard of 2.2 gallons. Construct a 99% confidence interval for σ^2, which measures the true variability of the gasoline consumption of the engine.

(b) Show that $Y = \frac{1}{6}(X_1 + 2X_2 + 3X_3)$ is not a sufficient estimator of the Bernoulli parameter θ.

10. State and prove the Neyman-Pearson Lemma.

11. Consider the following data on the number of hours that 10 persons studied for a French test and their scores on the test. Construct a 95% confidence interval for β.

Hours Studied x	4	9	10	14	4	7	12	22	1	17
Scores y	31	58	65	73	37	44	60	91	21	84

12. A car rental agency, which uses 5 different brands of tyres in the process of deciding the brand of tyre to purchase as standard equipment for its fleet, finds that each of 5 tyres of each brand last the following number of kilometers (in thousands).

Tyre Brands

A	B	C	D	E
36	46	35	45	41
37	39	42	36	39
42	35	37	39	37
38	37	43	35	35
47	43	38	32	38

Test the hypothesis that the five tyre brands have almost the same average life.

13. Evaluate the $\rho = 2$-variate normal density in terms of the individual parameters $\mu_1 = E(X_1), \mu_2 = E(X_2). \sigma_{11} = Var(X_1), \sigma_{22}$

$$= Var(X_2) and\, \rho_{12} = \frac{\sigma_{12}}{\sqrt{\sigma_{11}}\sqrt{\sigma_{22}}} = Corr(X_1, X_2).$$